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LETTER TO THE EDITOR

Gravitational wave dispersion in condensed matter systems

A Widom†, G Megaloudis†, T D Clark‡ and R J Prance‡

†Physics Department, Northeastern University, Boston, Massachusetts, USA
‡Physics Laboratory, University of Sussex, Brighton, Sussex, England

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Abstract. The complex index of gravitational wave refraction in condensed matter systems is related to the viscosity response function. Simple applications are discussed for gravitational wave propagation in fluids and crystals.

An engineering problem of considerable importance is the design of electro-mechanical systems which are sensitive enough to measure the deviations from Newtonian gravitational theory implied by general relativity (Gravitazione Sperimentale 1974). In previous work (Widom *et al* 1981), we considered the effects of static space–time metrics on superfluid flows. Here, we shall discuss gravitational wave metrics.

In the engineering design of electromagnetic wave detectors, it is useful to relate the complex index of refraction to the electrical conductivity (at complex frequency ζ) of the condensed matter. In terms of the dielectric response function this relation reads (Von Hippel 1966)

$$\varepsilon(\zeta) = 1 + (4\pi/\zeta)\mathrm{i}\sigma(\zeta). \tag{1}$$

Here, it will be shown that the complex index of gravitational wave refraction $N(\zeta)$ is related to the viscosity of the condensed matter $\eta(\zeta)$ via

$$N^2(\zeta) = 1 + (16\pi G/c^2\zeta)\mathrm{i}\eta(\zeta), \tag{2}$$

where G is the gravitational coupling constant and c is the speed of light.

To prove equation (2), we consider the metric of space–time in the transverse traceless gauge (Misner *et al* 1973), i.e. the gravitational wave is represented by a spatial non-Euclidean strain tensor

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - |\mathrm{d}\boldsymbol{r}|^2 - 2\,\mathrm{d}\boldsymbol{r}\cdot\boldsymbol{\psi}\cdot\mathrm{d}\boldsymbol{r}, \tag{3a}$$

$$\mathrm{div}\,\boldsymbol{\psi} = 0, \tag{3b}$$

$$\mathrm{Tr}\,\boldsymbol{\psi} = 0. \tag{3c}$$

The sources of the spatial gravitational strains are the transverse traceless parts of the spatial stresses (Foster and Nightingale 1979)

$$(\nabla^2 - \partial^2/c^2\,\partial t^2)\boldsymbol{\psi} = (8\pi G/c^4)\boldsymbol{\sigma}. \tag{4}$$

The constitutive equation which relates stress to strain in condensed matter systems defines a non-local viscosity (Landau and Lifshitz 1975) operator by

$$\boldsymbol{\sigma} = \hat{\eta}(2\partial\boldsymbol{\psi}/\partial t). \tag{5}$$

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For spatially isotropic condensed matter systems, and for wavelengths large on the length scale of atomic separations, equations (4) and (5) read in frequency space

$$(N^2(\zeta)\zeta^2 + c^2\nabla^2)\psi = 0, \tag{6}$$

which establishes $N(\zeta)$ in equation (2) as the proper gravitational wave index of refraction. A few applications will suffice.

An isotropic fluid of mass density ρ has a kinematic viscosity ν defined by

$$\rho\nu = \lim_{\omega\to 0} \mathrm{Re}\ \eta(\omega + \mathrm{i}0^+). \tag{7}$$

For an isotropic crystal it is more common to use the elastic Lamé coefficient response function

$$\mu(\zeta) = -\mathrm{i}\zeta\eta(\zeta). \tag{8}$$

Such crystals can support transverse sound waves at velocity v_t determined by

$$\rho v_t^2 = \lim_{\omega\to 0} \mathrm{Re}\ \mu(\omega + \mathrm{i}0^+) = \lim_{\omega\to 0} \omega\ \mathrm{Im}\ \eta(\omega + \mathrm{i}0^+). \tag{9}$$

We note, in passing, that if equations (1) and (2) for the square of the index of refraction are compared, then normal fluids act on gravitational waves analogously to the way that conductors act on electromagnetic waves. Further, crystals act on gravitational waves analogously to the way that superconductors act on electromagnetic waves. We shall return to this point, but here recall that the definition of the London penetration depth λ_L for static magnetic fields in terms of the electrical conductivity $\sigma(\zeta)$ is given by (Tinkham 1961)

$$(c/\lambda_L)^2 = 4\pi \lim_{\omega\to 0} \omega\ \mathrm{Im}\ \sigma(\omega + \mathrm{i}0^+) \tag{10}$$

and compare equations (9) and (10).

A gravitational wave will propagate with the speed of light in a viscous fluid,

$$\omega_Q = cQ, \tag{11}$$

as long as the damping rate

$$\gamma = (8\pi G\rho\nu/c^2) \tag{12}$$

is sufficiently small, $\gamma \ll \omega_Q$. In theory, there exists a strongly overdamped regime $\gamma \gg \omega_Q$ where the induced fluid stress modes cause the non-Euclidean spatial deformations to diffuse. The appropriate diffusion coefficient is given by

$$D = (c^2/2\gamma) = (c^4/16\pi G\rho\nu). \tag{13}$$

The gravitational wave index of refraction in a crystal when transverse sound waves propagate without damping is given by

$$N^2(\zeta) = 1 - (\Omega_0^2/\zeta^2), \tag{14}$$

where the gravitational wave 'plasma frequency' is determined as

$$\Omega_0^2 = (16\pi G\rho v_t^2/c^2). \tag{15}$$

Physically, equation (14) implies a gravitational wave dispersion relation of the form

$$\omega_Q^2 = c^2Q^2 + \Omega_0^2. \tag{16}$$

We now return to our analogy with electromagnetic waves within bulk superconductors; there, the dispersion relation is given by

$$\omega_Q^2 = c^2 Q^2 + (c/\lambda_L)^2, \tag{17}$$

where λ_L is the London penetration depth.

From the viewpoint of quantum gauge field theories (Weinberg 1965, Feynman 1963, De Witt 1967), equations (16) and (17) have the following interpretation: (i) equation (17) implies that a photon within a superconductor 'grows a mass'; (ii) equation (16) implies that a graviton within a crystal 'grows a mass'. The mechanism by which such Boson masses appear is a phase transition into a condensed matter state which breaks gauge symmetry by appropriately ordering. In the case of superconducting ordering, the induced current is proportional to the vector potential in the London–Coulomb gauge. In the case of crystal ordering, the induced stress is proportional to the spatial metric strain in the transverse traceless gauge (this is the evident meaning of elasticity coefficients). Given the analogy, it is clear that crystals should have a penetration depth in which induced stresses screen out gravitationally non-Euclidean spatial transverse metric strains. The penetration depth Λ is given by

$$\Lambda^{-2} = (16\pi G\rho v_t^2/c^4), \tag{18}$$

where ρ is the mass density and v_t is the transverse sound velocity within the crystal.

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